

Lecture 4: Intrinsic semiconductors

Contents

1	Introduction	1
2	Intrinsic Si	2
3	Conductivity equation	3
3.1	Electron mobility in Si	5
4	Carrier concentration in semiconductors	6
5	Intrinsic carrier concentration	9

1 Introduction

Semiconductors can be divided into two categories.

1. Intrinsic semiconductors
2. Extrinsic semiconductors

This classification is related to the purity of the semiconductors. *Intrinsic or pure semiconductors* are those that are ideal, with no defects, and no external impurities. The conductivity is temperature dependent. As opposed to intrinsic semiconductors, *extrinsic semiconductors* have some impurities added to modify the concentration of charge carriers and hence the conductivity. Extrinsic semiconductors are used extensively due to the ability to precisely tailor their conductivity by adding the impurities. Intrinsic semiconductors (especially Si and Ge) are used as optical and x-ray detectors (at low T) where a low concentration of charge carriers is required.

2 Intrinsic Si

Si is a semiconductor material with 4 electrons in the outer shell. These 4 electrons occupy 4 sp^3 hybrid orbitals with a tetrahedral arrangement. This gives rise to a full valence band (VB) and an empty conduction band (CB) at absolute zero with an energy gap of 1.17 eV between the two. This energy gap is called the *band gap*. This information is summarized in figure 1.

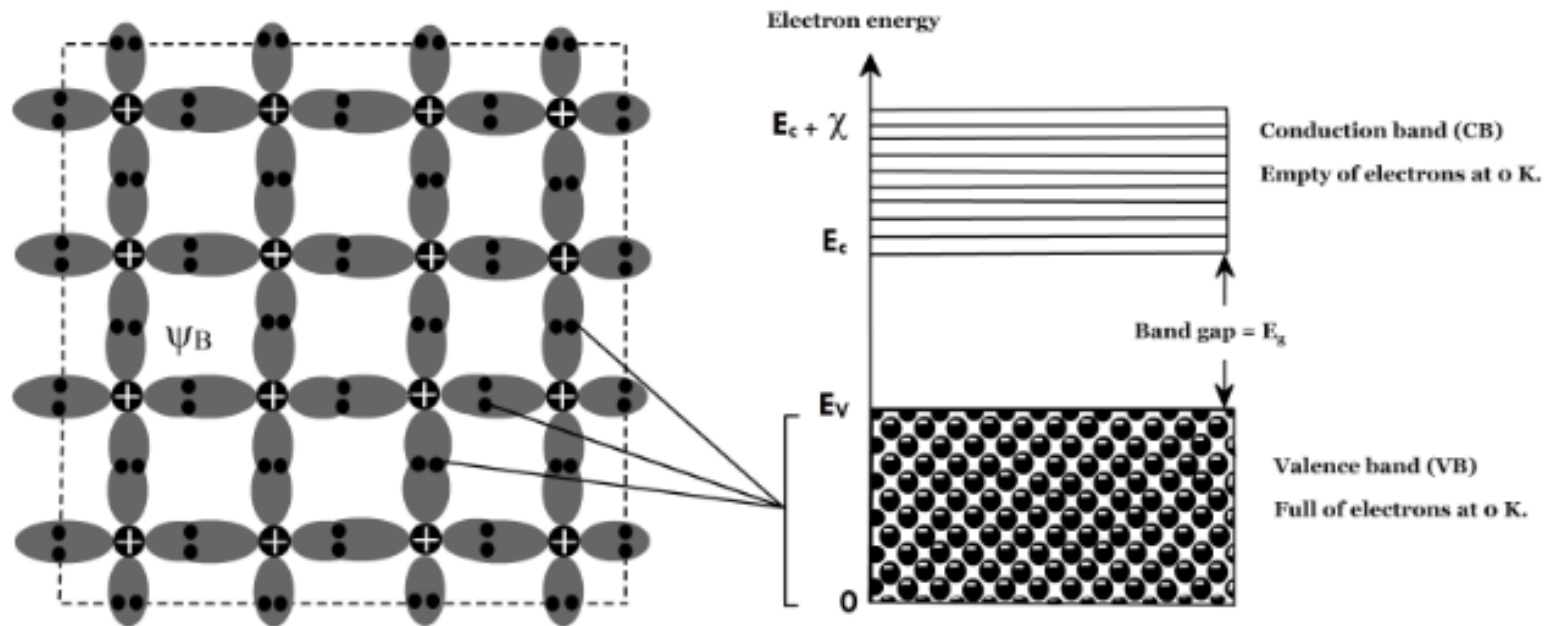


Figure 1: Hybrid orbitals in Si that forms a valence and conduction band with a band gap.

The reference is the bottom of the valence band. E_v and E_c represent the top of the valence band and the bottom of the conduction band. So that the energy distance between them is the band gap (E_g). The top of the conduction band is the vacuum level and is usually $E_c + \chi$ above the conduction band where χ is the *electron affinity* of Si. For Si, the electron affinity is 4.05 eV. The band gap at 0 K is 1.17 eV, while at room temperature the value is slightly lower, around 1.10 eV.

At any temperature, thermal excitation will always cause electrons to move from the valence band to the conduction band. These electrons in the conduction band are delocalized and can move in the solid by applying an electric field. Electrons in the conduction band leave behind holes in the valence band. These holes are also 'delocalized' and move in the direction opposite to the electrons. These electrons and holes are responsible for conduction. *Electron-hole pairs* in Si can also be generated by using electromagnetic radiation. The minimum energy of the radiation required is equal to the band gap. If the energy is less than E_g *electron excitation does not occur since there are no states in the band gap*. The relation between E_g and the maximum wavelength of excitation (λ) is given by

$$\lambda = \frac{hc}{E_g} = \frac{1240(eV \cdot nm)}{E_g(eV)} \quad (1)$$

$$h = 4.136 \times 10^{-15} \text{ eV} \cdot \text{s}$$

$$c = 3 \times 10^8 \text{ m/s} = 3 \times 10^{17} \text{ nm/s}$$

For Si, the wavelength is approximately 1060 nm and it lies in the IR region. This is the reason why Si is opaque since visible radiation (400 - 700 nm) will be absorbed by Si forming electron-hole pairs. Glass (SiO_2) on the other hand has a band gap of approximately 10 eV and hence the maximum wavelength is 100 nm (in the UV region). Absorption of light by Si is shown in figure 2.

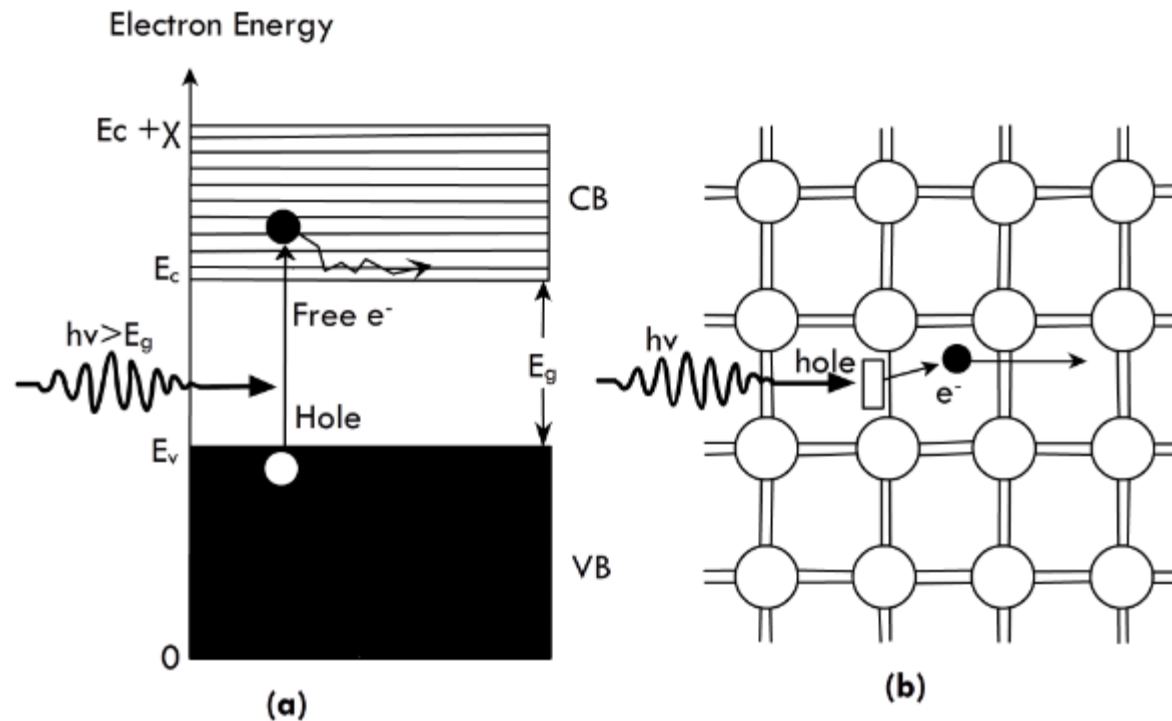
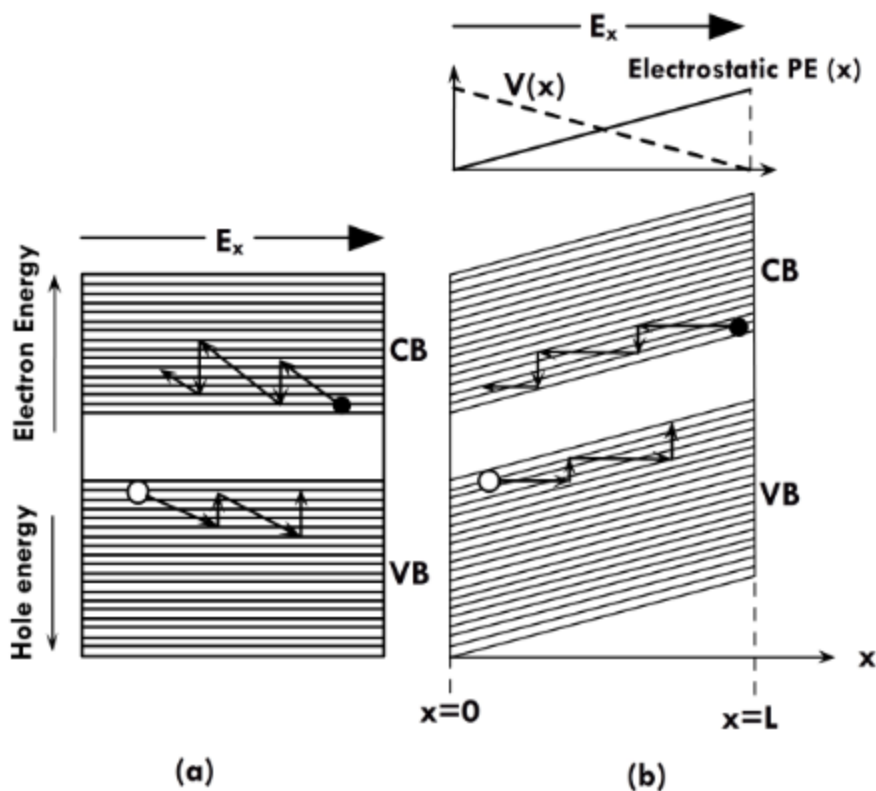


Figure 2: Electron-hole formation in Si due to absorption of light.

- (a) A photon with an energy greater than E_g can excite an electron from the VB to the CB.
- (b) When a photon breaks a Si-Si bond, a free electron and a hole in the Si-Si bond are created.

3 Conductivity equation

Conductivity in a semiconductor is due to movement of electrons in the CB and holes in the VB in an applied electric field. This is shown schematically in figure 3. These move in opposite directions since hole motion in the VB is due to electron motion in the opposite direction.



When an electric field is applied, electrons in the CB and holes in the VB can drift and contribute to the conductivity.

(a) A simplified illustration of drift in E_x .

(b) Applied field bends the energy bands since the electrostatic PE of the electron is $-eV(x)$ and $V(x)$ decreases in the direction of E_x , whereas PE increases.

Figure 3: Conduction in a semiconductor.

Figure 4 shows the hole motion in the VB due to electron tunneling from one bond to the next. Ultimately the hole recombines with the electron and gets annihilated. Thus formation of electron and holes and their recombination is a *dynamic process*. This depends on the temperature of the sample (for an intrinsic semiconductor) so that there is an equilibrium concentration at a given temperature.

Conductivity in a semiconductor depends on two factors

1. Concentration of electrons and holes. Denoted as n and p and is temperature dependent.
2. Ability of the electron and holes to travel in the lattice without scattering.

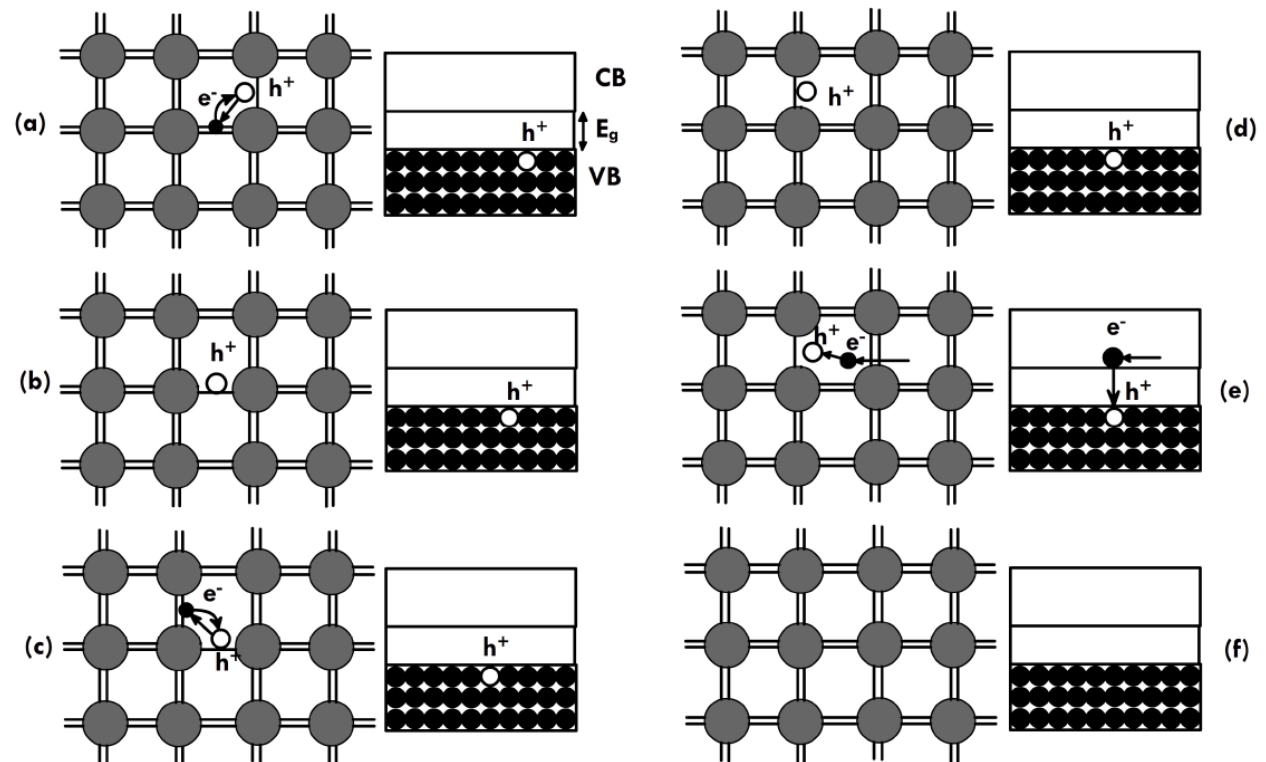


Figure 4: Hole motion and finally annihilation in a semiconductor.

Electrons and holes are said to *drift* in the lattice. This is because they undergo multiple scatterings with the atoms. For semiconductors concentration of electrons and holes are small so that electron-electron scattering can be ignored. Define a quantity called **mobility**, denoted by the symbol μ . Mobility refers to the ability of the carriers to move in the lattice. Mobility is related to the effective mass of the carrier (m_e^* or m_h^*) and the time between two scattering events (τ_e or τ_h). The relation is

$$\begin{aligned}\mu_e &= \frac{e\tau_e}{m_e^*} \\ \mu_h &= \frac{e\tau_h}{m_h^*}\end{aligned}\tag{2}$$

The effective mass term takes into account the effect of the lattice arrangement on the movement of the carriers. Using the carrier concentration and the concept of mobility it is possible to write a general equation for conductivity (σ) given by

$$\sigma = ne\mu_e + pe\mu_h\tag{3}$$

According to equation 3 higher the carrier concentration (n or p) higher the conductivity. Also, higher the mobility, higher the conductivity. Since mobility is related to the time between 2 scattering events, by equation 2, more the time between 2 scattering events greater is the conductivity.

3.1 Electron mobility in Si

Note transformation to m^2 in calculating τ

Consider the case of Si, where the electron mobility (μ_e) is $1350 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$ and μ_h is $450 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$. The effective masses are m_e^* is $0.26m_e$ and m_h^* is $0.38m_e$. Using equation 2 it is possible to find the scattering time for electrons and holes. Consider electrons, τ_e is calculated to be $2 \times 10^{-13} \text{ s}$ or 0.2 ps (pico seconds). This time, between 2 scattering events, is extremely short.

Mobility is temperature dependent. It also depends on the type of semiconductor and the presence of impurities. Mobility usually decreases with increasing impurity concentration since there are more scattering centers in the material.

Ge has a higher mobility than Si. μ_e for Ge is $3900 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$. μ_e for GaAs is even higher, $8500 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$. Thus, based on mobility, GaAs would be the material with the highest conductivity. But conductivity also depends on the carrier concentration. The dominating term, in equation 3, would determine the conductivity.

4 Carrier concentration in semiconductors

The carrier concentration in an energy band is related to the density of available states, $g(E)$, and the probability of occupation, $f(E)$. This is given by

$$n = \int_{band} g(E)f(E)dE \quad (4)$$

where the integration is over the entire band. $f(E)$ represents the Fermi function and for energy much greater than $k_B T$ it can be approximated by the Boltzmann function.

$$f(E) = \frac{1}{1 + \exp(\frac{E - E_F}{k_B T})} \simeq \exp[-\frac{(E - E_F)}{k_B T}] \quad (5)$$

To find the number of electrons in the conduction band (n) then equation 4 can be written as

$$n = \int_{E_c}^{E_c+\chi} g_{CB}(E)f(E)dE \quad (6)$$

where the simplified Fermi function is used. The actual density of states function in the CB depends on the semiconductor material but an approximation of a 3D solid with an uniform potential can be used. In this case $g_{CB}(E)$ turns out to be

$$g_{CB}(E) = \frac{8\pi\sqrt{2}}{h^3} (m_e^*)^{\frac{3}{2}} (E - E_c)^{\frac{1}{2}} \quad (7)$$

Since the density of states function is with respect to the bottom of the CB E is replaced by $(E - E_c)$. To further simplify the integral the limits can be changed from E_c to ∞ instead of $E_c + \chi$. This is because most of the electrons in the CB are close to the bottom and χ is usually much larger than $k_B T$. Hence making the substitutions in equation 6 and doing the integration the electron concentration in the conduction band is given by

$$\begin{aligned} n &= N_c \exp\left[-\frac{(E_c - E_F)}{k_B T}\right] \\ N_c &= 2\left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{\frac{3}{2}} \end{aligned} \quad (8)$$

where N_c is a temperature dependent constant called the *effective density of states at the conduction band edge*. It gives the total number of available states per unit volume at the bottom of the conduction band for electrons to occupy. E_c is the bottom of the conduction band and E_F is the position of the Fermi level.

A similar equation can be written for holes

$$\begin{aligned} p &= N_v \exp\left[-\frac{(E_F - E_v)}{k_B T}\right] \\ N_v &= 2\left(\frac{2\pi m_h^* k_B T}{h^2}\right)^{\frac{3}{2}} \end{aligned} \tag{9}$$

where N_v is the *effective density of states at the valence band edge*. Equations 8 and 9 give the electron and hole concentrations in semiconductors (intrinsic or extrinsic). The concentrations depend on the position of the Fermi level. The calculations are summarized in figure 5. This plots the DOS function in the CB and VB and the variation in the Fermi function in these bands. The Fermi level is usually far away from the band edges so that it can be approximated by the Boltzmann function. The electron and hole concentration is got by multiplying both and these are located close to the edge of the band.

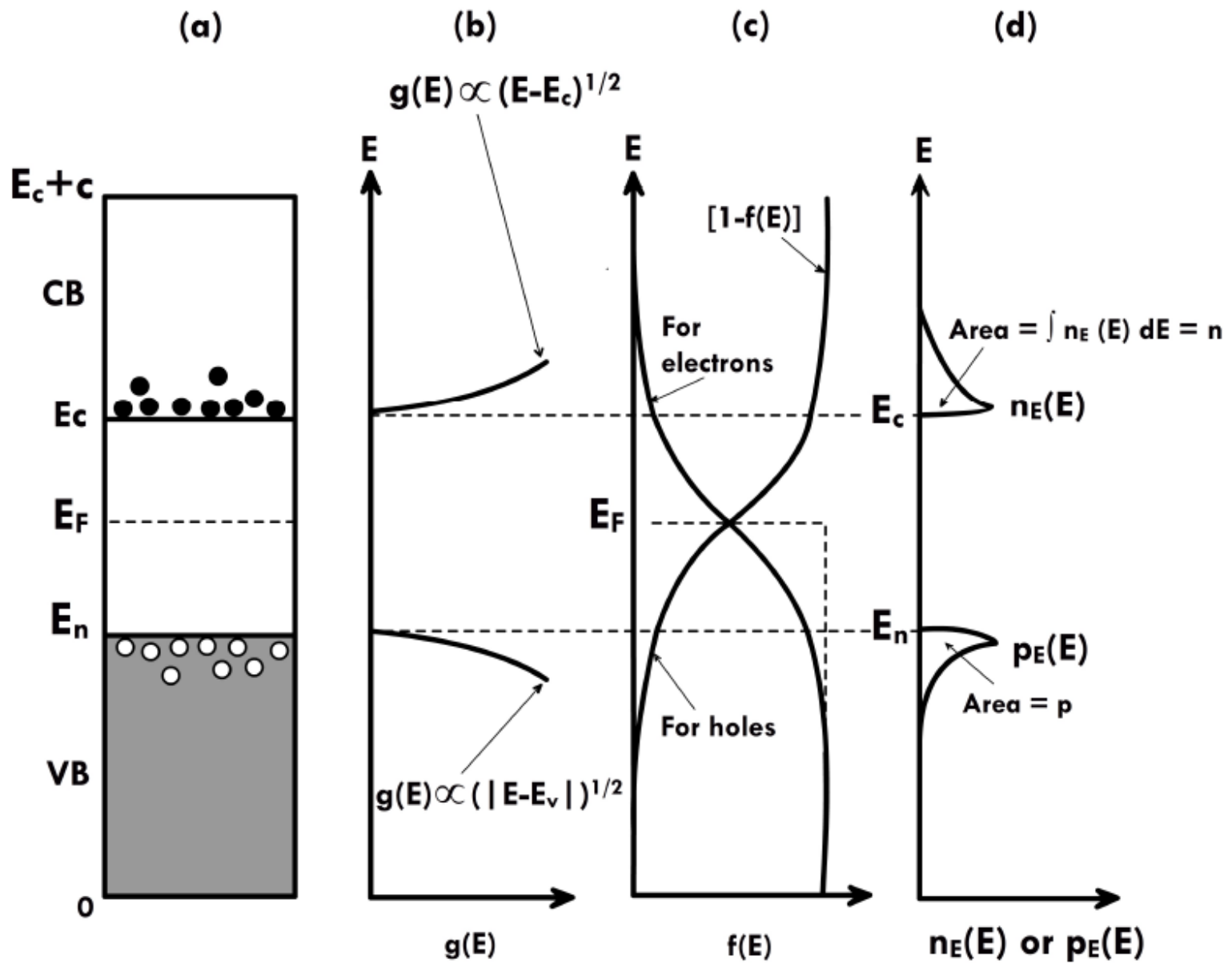


Figure 5: (a) Band picture of Si (b) DOS in CB and VB (c) Fermi function for electron and holes (d) Electron concentration in CB and hole concentration in VB.

5 Intrinsic carrier concentration

To eliminate E_F consider multiplying n and p in equations 8 and 9. This gives

$$np = N_c N_v \exp\left[-\frac{(E_c - E_v)}{k_B T}\right] \quad (10)$$

From figure 1 it can be seen that this is nothing but the band gap so that equation 10 becomes

$$np = N_c N_v \exp\left[-\frac{E_g}{k_B T}\right] \quad (11)$$

Thus, the product of electron and hole concentration is independent of the Fermi level position but only on the band gap and temperature, apart from N_c and N_v . In an intrinsic semiconductor $n = p$ since electron and holes are created in pairs (hole is the absence of electron). This is called n_i the **intrinsic carrier concentration**. Substituting in equation 11 the intrinsic carrier concentration can be calculated.

$$n_i = \sqrt{N_c N_v} \exp\left(-\frac{E_g}{2k_B T}\right) \quad (12)$$

Thus, the intrinsic carrier concentration of a semiconductor is dependent only on the band gap (E_g). n_i is a material property (at a given temperature). Equation 11 can now be rewritten as

$$\boxed{np = n_i^2} \quad (13)$$

This equation is called **law of mass action** and it is valid for any semiconductor at equilibrium. For an intrinsic semiconductor equation 13 is trivial since $n = p = n_i$ but even when n and p are not equal to product should still yield n_i^2 . This has important implications for extrinsic semiconductors. The conductivity equation 3 can now be rewritten for intrinsic semiconductors as

$$\sigma_i = n_i e (\mu_e + \mu_h) \quad (14)$$

EXAMPLE

INTRINSIC CONCENTRATION AND CONDUCTIVITY OF Si Given that the density of states related effective masses of electrons and holes in Si are approximately $1.08m_e$ and $0.60m_e$, respectively, and the electron and hole drift mobilities at room temperature are 1350 and $450 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$, respectively, calculate the intrinsic concentration and intrinsic resistivity of Si.

SOLUTION We simply calculate the effective density of states N_c and N_v by

$$N_c = 2 \left(\frac{2\pi m_e^* kT}{h^2} \right)^{3/2} \quad \text{and} \quad N_v = 2 \left(\frac{2\pi m_h^* kT}{h^2} \right)^{3/2}$$

Thus

$$N_c = 2 \left[\frac{2\pi (1.08 \times 9.1 \times 10^{-31} \text{ kg})(1.38 \times 10^{-23} \text{ J K}^{-1})(300 \text{ K})}{(6.63 \times 10^{-34} \text{ J s})^2} \right]^{3/2}$$

$$= 2.81 \times 10^{25} \text{ m}^{-3} \quad \text{or} \quad 2.81 \times 10^{19} \text{ cm}^{-3}$$

and

$$N_v = 2 \left[\frac{2\pi (0.60 \times 9.1 \times 10^{-31} \text{ kg})(1.38 \times 10^{-23} \text{ J K}^{-1})(300 \text{ K})}{(6.63 \times 10^{-34} \text{ J s})^2} \right]^{3/2}$$

$$= 1.16 \times 10^{25} \text{ m}^{-3} \quad \text{or} \quad 1.16 \times 10^{19} \text{ cm}^{-3}$$

The intrinsic concentration is $n_i = (N_c N_v)^{1/2} \exp\left(-\frac{E_g}{2kT}\right)$

$$n_i = [(2.81 \times 10^{19} \text{ cm}^{-3})(1.16 \times 10^{19} \text{ cm}^{-3})]^{1/2} \exp\left[-\frac{(1.10 \text{ eV})}{2(300 \text{ K})(8.62 \times 10^{-5} \text{ eV K}^{-1})}\right]$$

$$= 1.0 \times 10^{10} \text{ cm}^{-3}$$

The conductivity is $\sigma = en\mu_e + ep\mu_h = en_i(\mu_e + \mu_h)$

$$\text{that is, } \sigma = (1.6 \times 10^{-19} \text{ C})(1.0 \times 10^{10} \text{ cm}^{-3})(1350 + 450 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1})$$

$$= 2.9 \times 10^{-6} \Omega^{-1} \text{ cm}^{-1}$$

The resistivity is $\rho = \frac{1}{\sigma} = 3.5 \times 10^5 \Omega \text{ cm}$

Table 1. is a comparative table of some of the properties of the important semiconductors, Ge, Si, and GaAs.

Selected typical properties of Ge, Si, and GaAs at 300 K

	E_g (eV)	χ (eV)	N_c (cm ⁻³)	N_v (cm ⁻³)	n_i (cm ⁻³)	μ_e (cm ² V ⁻¹ s ⁻¹)	μ_h (cm ² V ⁻¹ s ⁻¹)	m_e^*/m_e	m_h^*/m_e	ϵ_r
Ge	0.66	4.13	1.04×10^{19}	6.0×10^{18}	2.3×10^{13}	3900	1900	0.12a 0.56b	0.23a 0.40b	16
Si	1.10	4.01	2.8×10^{19}	1.2×10^{19}	1.0×10^{10}	1350	450	0.26a 1.08b	0.38a 0.60b	11.9
GaAs	1.42	4.07	4.7×10^{17}	7×10^{18}	2.1×10^6	8500	400	0.067a,b 0.50b	0.40a	13.1

NOTE: Effective mass related to conductivity (labeled a) is different than that for density of states (labeled b). In numerous textbooks, n_i is taken as $1.45 \times 10^{10} \text{ cm}^{-3}$ and is therefore the most widely used value of n_i for Si, though the correct value is actually $1.0 \times 10^{10} \text{ cm}^{-3}$. (M. A. Green, *J. Appl. Phys.*, **67**, 2944, 1990.)